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Full Length Article

Comparison of rock spalling evaluation in underground openings: Uncertainty-based mathematical model and empirical method

Chao Chen [a], Shibin Yao [b], Jian Zhou [b,*]

[a] School of Civil and Hydraulic Engineering, Huazhong University of Science and Technology, 1037 Luoyu Road, Hongshan District, Wuhan, Hubei 430074, China
[b] School of Resources and Safety Engineering, Central South University, Changsha 410083, China

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ABSTRACT

Rock spalling, considered an underground geological hazard that usually occurs after excavation in hard rock, threatens the safety of on-site operators and equipment. This study used an objective uncertainty-based model integrating six membership functions to evaluate rock spalling. Meanwhile, a new index, spalling coefficients, as a robust empirical method referring to the rich engineering experience of researchers, was proposed for spalling risk evaluation. Then, 60 groups of the spalling dataset are collected from the literature to validate the performance of the proposed data-driven model and spalling coefficients. The results reveal that the constructed uncertainty model and spalling coefficients method can evaluate the spalling properly from different terms, which can be considered a valuable theoretical tool for underground hazard control and prevention.

1. Introduction

Various geological hazards associated with rock failures occur more frequently underground as the expansion of excavation engineering at great depths [1]. It is widely accepted that rock mass property and in situ stress are the main factors contributing to the failures of underground infrastructures [2,3]. Generally, these rock failures can be summarized in two categories [4]: gravity-induced failure and stress-induced failure. Rock spalling, as one type of rock burst, can be produced in hard rock after the target area is excavated and stress redistribution. This hazard generally causes severe damage to surrounding rock and casualties. It is, therefore, significant to explore some highly efficient models/criteria to evaluate rock spalling, e.g., numerical simulation models [5–7], mathematical models [8–11], laboratory tests [12,13] and various quantitative and qualitative evaluation criteria [14,15] etc. Panthi et al. [16] explicitly discussed the application scenarios of multiple methods, including the Norwegian rule of thumb, stress problem classification, uniaxial compressive and tensile strength approach, and rock spalling strength approach. They pointed out that most require a full understanding of in situ stress for the target area. Mathematical and empirical models are often the go-to tools for researchers when addressing new problems. However, due to the subjectivity of empirical models and the lack of sufficient data to support them, they are typically only able to solve a limited range of problems in specific areas. On the other hand, while mathematical models rely on various assumptions when solving problems, these assumptions can often lead to the omission of high-impact factors, which in turn may compromise the model's effectiveness.

Numerical simulation methods play an important role in rock burst evaluation/prediction with the fast development of advanced computer science. However, it also suffers from some shortcomings: 1) the prediction accuracy largely depends on the size of calibrated samples used; 2) the selection of input parameters and interpretation of results need to be considered. 3) as a direct and simple theoretical tool, empirical criteria can roughly evaluate the risk of rock spalling. It is usually constructed based on local area rather than large-scale on-site investigation [17], and only a few influencing factors are considered, resulting in the final evaluation results' low accuracy. In addition, due to the current limitations of scientific knowledge, rock engineering is often influenced by various natural environmental factors (such as geological conditions, climate change, groundwater, etc.) and artificial factors (such as design, construction, and mining activities). These factors provide a significant amount of uncertain information for decision-makers and the development of evaluation models.

A briefly bibliometric analysis in Fig. 1 is conducted to visualize the developed level of spalling research in this field. A topic search strategy with the term “spalling”, while the Categories are set as “Engineering Geological” AND “Geosciences Multidisciplinary” AND “Mining Mineral

* Corresponding author.
E-mail addresses: chao_chen_best@hotmail.com (C. Chen), shibin.yao@csu.edu.cn (S. Yao), j.zhou@csu.edu.cn (J. Zhou).

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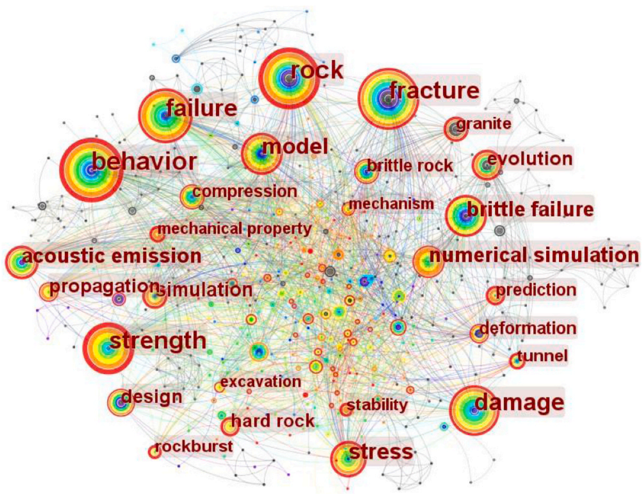

Fig. 1. Keywords analysis regarding rock spalling research.

Processing" AND "Geology" AND "Engineering Petroleum". The top ten keywords are "rock", "behavior", "strength", "fracture", "damage", "failure", "stress", "model", "brittle failure" and "numerical simulation", corresponding to the frequency of 56, 55, 54, 52, 46, 46, 36, 36, 29 and 26, respectively. It can be seen that a large volume of research focuses on the mechanism and rock behaviour while analyzing this geological hazard; that is, few concerns have been attached to spalling from the perspective of the evaluation model considering uncertainty factors.

Many engineering projects are affected by uncertainty [18]. For example, the number and distribution of joints in rock masses cannot be accurately simulated, but we can still analyze rock mass stability from other perspectives. Thus, we are expected to accurately evaluate rock spalling based on available data collected rather than considering all geotechnical engineering factors. Assuming that unascertained uncertainty is the main issue affecting the model performance, the most reasonable approach is incorporating uncertainty information directly. In recent years, uncertainty theory has been applied in the rock engineering field, e.g., rock burst [19], geotechnical engineering [20,21] and rock fragmentation [22]; the classification results demonstrate its applicability effectively.

In summary, traditional research on rock spalling evaluation has often focused more on obtaining deterministic results while neglecting the uncertainties inherent in model construction. This inevitably leads to the premature determination of a limited set of variables, selectively overlooking potentially crucial factors that ultimately limit the scalability and reliability of the evaluation model.

This paper proposes a hybrid model based on unascertained measurement and an objective weight method [23,24]. By directly incorporating uncertainty information, the model reduces information loss and enables a more accurate evaluation of rock spalling. This model reduces uncertainty in engineering and enhances the reliability of initial engineering designs, offering a more robust and comprehensive method for evaluating rock spalling. Section 2 introduces the research methods used in this study and summarises concepts such as the membership function. Section 3 applies the research methods to engineering practice. In Section 4, we discuss the research results. Section 5 summarizes the conclusions of the study.

2. Method

Assume U and F are the object space $U = \{X, X_2, \cdots, X_n\}$ and the state space, respectively. E is the topology structure of F. $I = \{I_1, I_2, \cdots, I_m\}$ represents the set of evaluation indicators. The vector $X_{ij}(i = 1, 2, \cdots, n;\ j = 1, 2, \cdots, m)$ is the initial value of sample X_i under index I_j. The observed matrix is shown as follows [18]:

$$X = \begin{Bmatrix} x_{11} & x_{12} & \cdots & x_{1m} \\ x_{21} & x_{22} & \cdots & x_{2m} \\ \cdots & \cdots & \cdots & \cdots \\ x_{n1} & x_{n2} & \cdots & x_{nm} \end{Bmatrix} \tag{1}$$

Definition 1. Measurable space (F, E) consists of the nature space F and the topology structure E. For $\forall x \in U, A \in E$, there are three principles that the degree $\mu_A(x)$ must be meet simultaneously [22]:

(1) Non-negative and limited principle: $\forall x \in U, \forall x \in E,\ 0 \leqslant \mu_A(x) \leqslant 1$.
(2) Additive principle: $\forall x \in U, \forall x \in E, A_i \cap A_j = \phi$, then

$$\mu_{\cup A}(x) = \sum\nolimits_j \mu_{Aj}(x) \tag{2}$$

(3) Convergent principle: for $\forall x \in U, \mu_F(x) = 1$.

Definition 2. $\mu_A(x)$ is the membership function while the three principles above are met simultaneously.

Definition 3. $(F, E, \mu_A(x))$ is defined as the unascertained measure and C is the grade vector $C_d(d = 1, 2, \cdots, p)$.

2.1. Construction of the membership function

Uncertainty measurement is a mathematical method used for quantitative analysis and decision-making under incomplete information and partially unknown circumstances [25,26]. It primarily deals with situations where the information entropy is neither completely random nor completely fuzzy but rather partially known yet imprecise. This method can help mitigate the challenges of data scarcity and strong subjectivity encountered during evaluation. The membership function is the function that is used to quantitatively describe the membership of individual indexes belonging to corresponding categories. The membership function of unascertained measurement theory meets the three principles of measure simultaneously. The membership function is a single variable function of the target set, and its essence is the determination of the unascertained function, which refers to the prior knowledge. The linear membership function is the most commonly used in previous studies. However, most rock engineering issues suffer from uncertainty and show non-linear characteristics. This work applies several non-linear functions to describe the complex behaviours of rock mass characteristics in real situations. The details of these membership functions are listed as follows:

(1) Linear function (Lin)

$$\begin{cases} \mu_i(x) = \begin{cases} \dfrac{-x}{right_{i+1} - left_i} + \dfrac{right_{i+1}}{right_{i+1} - left_i} & left_i < x \leqslant right_{i+1} \\ 0 & x > right_{i+1} \end{cases} \\ \mu_{i+1}(x) = \begin{cases} 0 & x \leqslant left_i \\ \dfrac{x}{right_{i+1} - left_i} - \dfrac{left_i}{right_{i+1} - left_i} & left_i < x \leqslant right_{i+1} \end{cases} \end{cases} \tag{3}$$

(2) Parabolic function (Para)

$$\begin{cases} \mu_i(x) = \begin{cases} 1 - \left(\dfrac{x - left_i}{right_{i+1} - left_i}\right)^2 & left_i < x \leqslant right_{i+1} \\ 0 & x > right_{i+1} \end{cases} \\ \mu_{i+1}(x) = \begin{cases} 0 & x \leqslant left_i \\ \left(\dfrac{x - left_i}{right_{i+1} - left_i}\right)^2 & left_i < x \leqslant right_{i+1} \end{cases} \end{cases} \tag{4}$$

(3) Exponential function (Exp)

$$\begin{cases} \mu_i(x) = \begin{cases} 1 - \dfrac{1 - \exp(x - left_i)}{1 - \exp(right_{i+1} - left_i)} & left_i < x \le right_{i+1} \\ 0 & x > right_{i+1} \end{cases} \\ \mu_{i+1}(x) = \begin{cases} 0 & x \le left_i \\ \dfrac{1 - \exp(x - left_i)}{1 - \exp(right_{i+1} - left_i)} & left_i < x \le right_{i+1} \end{cases} \end{cases} \tag{5}$$

(4) Sine function (Sin)

$$\begin{cases} \mu_i(x) = \begin{cases} \dfrac{1}{2} - \dfrac{1}{2}\sin\left\{\dfrac{\pi}{right_{i+1} - left_i}\left(x - \dfrac{right_{i+1} - left_i}{2}\right)\right\} & left_i < x \le right_{i+1} \\ 0 & x > right_{i+1} \end{cases} \\ \mu_{i+1}(x) = \begin{cases} 0 & x \le left_i \\ \dfrac{1}{2} + \dfrac{1}{2}\sin\left\{\dfrac{\pi}{right_{i+1} - left_i}\left(x - \dfrac{right_{i+1} - left_i}{2}\right)\right\} & left_i < x \le right_{i+1} \end{cases} \end{cases} \tag{6}$$

(5) Logarithmic function (Log)

$$\begin{cases} \mu_i(x) = \begin{cases} 1 - \log_2^{\left(\frac{x - left_i}{right_{i+1} - left_i} + 1\right)} & left_i < x \le right_{i+1} \\ 0 & x > right_{i+1} \end{cases} \\ \mu_{i+1}(x) = \begin{cases} 0 & x \le left_i \\ \log_2^{\left(\frac{x - left_i}{right_{i+1} - left_i} + 1\right)} & left_i < x \le right_{i+1} \end{cases} \end{cases} \tag{7}$$

(6) S function (S)

$$\begin{cases} \mu_i(x) = \begin{cases} 1 & x \le left_i \\ 1 - 2\left(\dfrac{x - left_i}{d - left_i}\right)^2 & left_i < x \le \dfrac{left_i + right_{i+1}}{2} \\ 2\left(\dfrac{x - right_{i+1}}{right_{i+1} - left_i}\right)^2 & \dfrac{left_i + right_{i+1}}{2} < x < right_{i+1} \\ 0 & x \ge right_{i+1} \end{cases} \\ \mu_{i+1}(x) = \begin{cases} 0 & x \le left_i \\ 2\left(\dfrac{x - left_i}{right_{i+1} - left_i}\right)^2 & left_i < x \le \dfrac{left_i + right_{i+1}}{2} \\ 1 - 2\left(\dfrac{x - right_{i+1}}{right_{i+1} - left_i}\right)^2 & \dfrac{left_i + right_{i+1}}{2} < x < right_{i+1} \\ 1 & x \ge right_{i+1} \end{cases} \end{cases} \tag{8}$$

(7) Weibull function (W)

$$\begin{cases} \mu_i(x) = \begin{cases} 1 - \dfrac{e}{e-1}\left(1 - \exp\left(-\left(\dfrac{x - left_i}{right_{i+1} - left_i}\right)\right)\right) & left_i < x \le right_{i+1} \\ 0 & x > right_{i+1} \end{cases} \\ \mu_{i+1}(x) = \begin{cases} 0 & x \le left_i \\ \dfrac{e}{1-e}\left(1 - \exp\left(-\left(\dfrac{x - left_i}{right_{i+1} - left_i}\right)\right)\right) & left_i < x \le right_{i+1} \end{cases} \end{cases} \tag{9}$$

where, $\mu_i(x)$ and $\mu_{i+1}(x)$ are the membership degree of index associated with grade C_d and C_{d+1}, respectively. $left_i$ and $right_{i+1}$ are specified as the endpoints of interval i.

2.2. Determination of weights

Calculation of index weight is one of the important procedures for comprehensive evaluation. Generally, the methods/theories used include analysis hierarchy process, brainstorming, CRITIC method, etc. Here, the entropy weight method is introduced to determine the index importance comprehensively considering the computation cost and objectivity.

The general methods used to calculate the criterion weight include the Delphi method, Brainstorming, AHP, etc.

Here, the index weights are determined using the entropy weight method. The entropy weight method reflects the degree of dispersion/uncertainty of the information contained in each indicator. The more dispersed the information of an indicator, the larger the entropy, and its corresponding weight will be smaller. Conversely, the weight will be

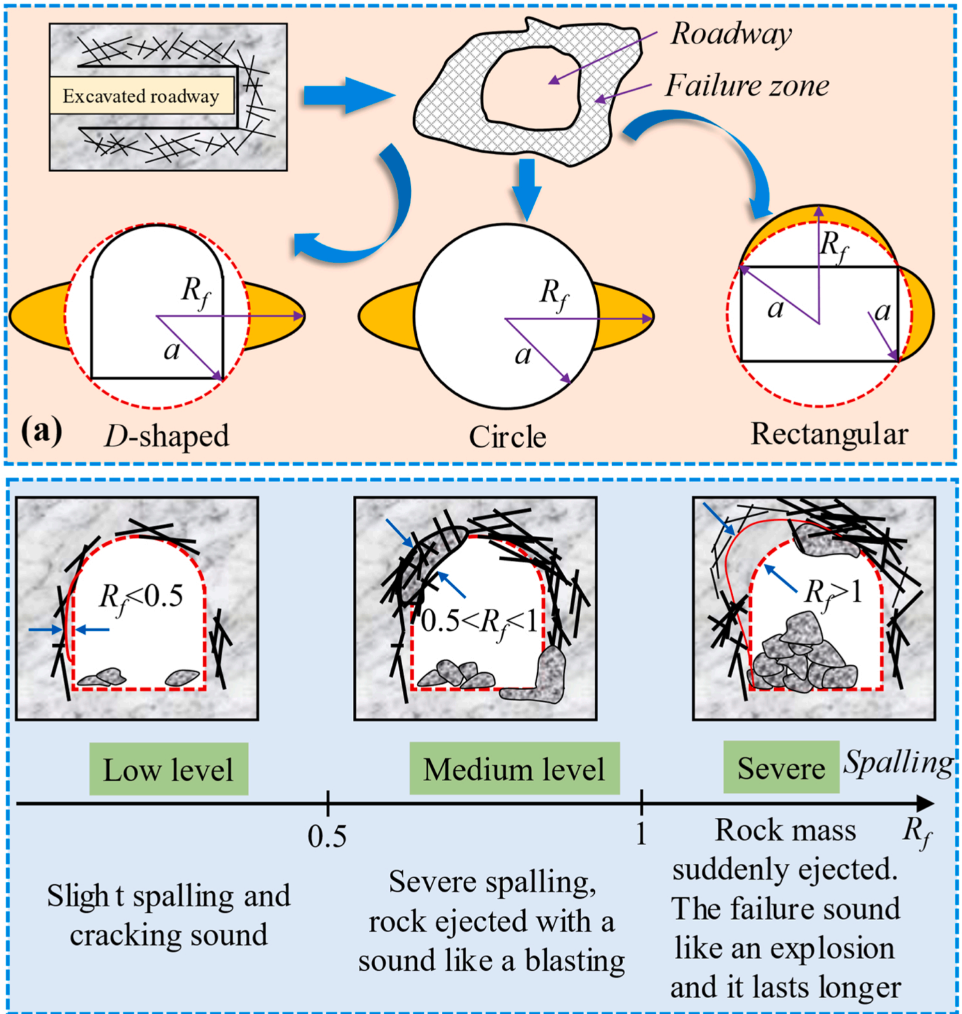

Fig. 2. Classification of rock spalling grades (Modified from [2–4]).

larger when the entropy is smaller. The principle of this theory is outlined in Eq. (10):

$$\chi_{ij} = 1 + \frac{1}{\ln p}\sum_{d=1}^{p} \mu_{ijd} \ln \mu_{ijd}, (1 \le i \le n; 1 \le j \le m; 1 \le d \le p) \tag{10}$$

$$\omega_{ij} = \frac{\chi_{ij}}{\sum_{j=1}^{m} \chi_{ij}} (1 \le i \le n; 1 \le j \le m) \tag{11}$$

where, χ_{ij} is the entropy value of index j, μ_{ijd} is the index measurement vector of jth index under ith sample corresponding to grade C_d, the weight coefficients ω_{ij} can be obtained.

2.3. Measurement vectors

The measurement vectors can be calculated after the multiple index measurement matrix and index weights are determined, reflecting the degree of samples belonging to each grade, which can further judge the risk level of samples. The calculation is given as Eq. (12):

$$Z_{id} = \sum_{j=1}^{m} \omega_{ij} \mu_{ijd} (1 \le i \le n; 1 \le j \le m; 1 \le d \le p) \tag{12}$$

Where, Z_{id} represents the comprehensive measurement of sample i belonging to grade C_d, μ_{ijd} and ω_{ij} are connected to single measurement vector and weight coefficient, respectively.

2.4. Spalling evaluation criterion

For the grade vector C_i, it meets $C_1 < C_2 \cdots C_{p-1} < C_p$ or $C_1 > C_2 \cdots C_{p-1} > C_p$. The shortcoming of maximum measure principle is obvious [1], the credible identification principle is used to evaluate the sample grade.

$$\phi_{id} = \min\left(d : \sum_{d=1}^{p} Z_{id} \ge R_\eta\right), (1 \le i \le n; 1 \le d \le p) \tag{13}$$

where, ϕ_{id} represents the evaluation grade of ith sample, while R_η is the credible identification principle.

The value of R_η varies from 0.5 to 1 according to the attribute identification theoretical model proposed by Cheng [27,28]. Commonly, R_η was set as 0.5–0.7 in other studies [22].

2.5. Sample sortation

The superiority of the dataset is calculated based on Eq. (14), through which the risk level of the individual sample can be observed clearly. The principle of procedure assigns the value for different grades, and then the risk score of samples can be obtained.

Table 1
Initial data associated with spalling evaluation [32].

No.	R_f/m	a/m	σ_1/Mpa	σ_3/Mpa	UCS/Mpa	σ_{cm}	σ_1/σ_3	Grade	Empirical method
1	3.42	2.63	29.38	15.3	100	16.67	1.92	3	3
2	3.95	2.63	30.64	14.8	100	16.67	2.07	3	3
3	3.68	2.63	29.84	14.7	100	16.67	2.03	3	3
4	3.95	2.63	34.23	16.3	100	16.67	2.1	3	3
5	3.95	2.63	31.26	15.4	100	16.67	2.03	3	3
6	4.21	2.63	33.02	15.8	100	16.67	2.09	3	3
7	2.09	1.16	139.75	65	350	109.13	2.15	3	3
8	1.97	1.16	139.75	65	350	109.13	2.15	3	3
9	1.62	1.16	111.6	60	350	109.13	1.86	3	3
10	1.74	1.16	111.6	60	350	109.13	1.86	3	3
11	1.65	1.18	52.55	15.5	250	65.88	3.39	3	3
12	1.53	1.18	52.55	15.5	250	65.88	3.39	3	3
13	2.63	1.75	58.96	11	220	54.39	5.36	3	3
14	2.45	1.75	58.96	11	220	54.39	5.36	3	3
15	2.45	1.75	58.96	11	220	54.39	5.36	3	3
16	2.28	1.75	58.96	11	220	54.39	5.36	3	3
17	2.28	1.75	58.96	11	220	54.39	5.36	3	3
18	1.75	1.75	40.7	11	220	54.39	3.7	3	2
19	2.53	2.3	52.4	40	220	54.39	1.31	3	3
20	5.32	3.8	10	5	36	3.6	2	3	3
21	5.5	5	15.73	12.1	80	11.93	1.3	3	2
22	1.07	1.07	35.49	21	217	53.28	1.69	3	2
23	1.16	1.07	33.8	20	151	30.93	1.69	3	3
24	5.3	3.8	19.5	8.1	150	30.62	2.41	3	2
25	3.85	3.4	20.2	10.1	175	38.58	2	3	1
26	6.8	4.8	31.3	10.4	195	45.38	2.99	3	2
27	7.5	6.7	67.72	44.01	112.5	19.89	1.54	3	3
28	6.4	6.2	64.45	41.58	112.5	19.89	1.55	3	3
29	7.2	6.2	62.9	44.7	112.5	19.89	1.41	3	3
30	6.39	4.59	54.52	40.62	99.6	16.56	1.34	3	3
31	5.39	4.59	52.09	39.04	112.5	19.89	1.33	3	3
32	6.26	4.96	53.92	40.85	112.5	19.89	1.32	3	3
33	5.76	4.96	68.98	43.83	112.5	19.89	1.57	3	3
34	4.5	3.6	59.51	39.3	136.8	26.67	1.51	3	3
35	8.6	6.2	56.84	38.79	112.5	19.89	1.47	3	3
36	6.2	5.6	28.7	7.95	120	21.91	3.61	3	3
37	5.4	5.25	20	2	200	47.14	10	3	1
38	1.12	1.07	45	20	76	11.04	2.25	3	3
39	3.38	2.98	24.5	7.5	154	31.85	3.27	3	2
40	4.99	4.69	24	12	109.5	19.1	2	3	3
41	2.05	1.85	48	14	160	33.73	3.43	3	3
42	4.27	3.27	51	31	104.5	17.8	1.65	3	3
43	3.06	2.86	55	28	230	58.14	1.96	3	3
44	10.6	10.2	16.8	4.5	94	15.19	3.73	3	2
45	5.3	4.7	38	9	200	47.14	4.22	3	2
46	2.1	1.85	36.72	10.02	126.5	23.71	3.66	3	3
47	2.05	1.85	36.72	10.02	126.5	23.71	3.66	3	3
48	2.45	1.85	36.72	10.02	126.5	23.71	3.66	3	3
49	4	2.83	35	23.4	100	16.67	1.5	3	3
50	4.01	3.01	60	43	240	61.97	1.4	3	3
51	2.53	2.5	32	10	195	45.38	3.2	3	2
52	0.96	0.88	30	10	211	51.08	3	2	2
53	5.25	4.85	21.2	4.5	80	11.93	4.71	3	3
54	5.35	4.85	21.2	4.5	80	11.93	4.71	3	3
55	5.05	4.85	31.16	15.07	129.9	24.68	2.07	3	3
56	5	4.85	31.16	15.07	142.4	28.32	2.07	3	3
57	5.75	5.4	25.86	7.23	53	6.43	3.58	3	3
58	5.8	5.4	25.86	7.23	53	6.43	3.58	3	3
59	6.9	5.4	25.86	7.23	25.1	2.1	3.58	3	3
60	7.15	5.4	25.86	7.23	25.1	2.1	3.58	3	3

$$A_d^i = \sum_{d=1}^{p} F_d Z_{id}, (1 \le i \le n; 1 \le d \le p) \tag{14}$$

The determination F_d is in the decision-makers hands (ascending or descending order). A_d^i is the score of sample i.

2.6. Empirical method for spalling evaluation

Assume $U = \{X, X_2, \cdots, X_n\}$ and $I = \{I_1, I_2, \cdots, I_m\}$ are the sample set and index system in the comprehensive evaluation, respectively. Simultaneously, assume $F = \{f_1, f_2, \cdots, f_m\}$ and $S = \{s_1, s_2, \cdots, s_m\}$ are the rating of the individual index and default value corresponding to each grade, respectively. Then, the empirical model can be constructed by integrating the index score and sample spalling coefficients as follows:

$$IS_i = \sum_{j=1}^{m} f_j s_j \tag{15}$$

$$SS_i = \frac{IS_i}{TS} \tag{16}$$

Table 2
Classification standard for rock spalling evaluation.

	σ_1/σ_c	Di	σ_{dev}/σ_{cm}	Rating
Low	< 0.1	< 0.3	< 0.8	0
Medium	0.1–0.2	0.3–0.5	0.8–1	1
Strong	> 0.2	> 0.5	> 1	2
Empirical score	70	30	40	

$$TS = \sum_{j=1}^{m} f_j^G s_j \tag{17}$$

where, SS_i is the risk coefficient referring to sample i, which can be calculated through individual sample score IS_i and sample maximum score TS.

3. Engineering application

3.1. Background descriptions

Underground rock failures are generally induced by high geo-stress and rock mass property, resulting in casualties and infrastructure damage [29]. Rock failures can be categorized as those induced by gravity or stress redistribution. Local stress concentration is the main factor that causes the spalling of rock mass, while the excavation objective is hard rock. Rock spalling is a common progressive destruction process in underground infrastructure, severely threatening the stability of rock-related frameworks. It can develop peacefully or change into severe rock bursts, usually at great depth [30]. The schematic diagram and its classification criteria regarding excavated rock spalling are shown in Fig. 2.

The stress change path of loading and unloading on surrounding rock is considerably complex while excavating the rock mass underground [31]. Deformation mechanism analysis regarding rock mass is currently hard to operate and does not meet on-site analysis requirements. With the rapid development of computer sciences, comprehensive evaluation incorporating multiple criteria is considered a highly efficient method. In this study, 60 groups of rock spalling records [32] are collected from the literature. Simultaneously, the ratio of maximum stress to the uniaxial compressive strength, the damage index and deviatoric stress to in situ stress strength are determined as the basic indicators for rock spalling evaluation. The calculation for evaluation indexes is shown as following [4,33]:

$$D_i = \frac{\sigma_{\max}}{\sigma_c} = \frac{3\sigma_1 - \sigma_3}{\sigma_c} \tag{18}$$

$$\frac{\sigma_{dev}}{\sigma_{cm}} = \frac{\sigma_1 - \sigma_3}{\sigma_{cm}} \tag{19}$$

Where, $\sigma_{\max}$ and σ_{dev} are specified as the tangential maximum and deviatoric stress, while σ_c and σ_{cm} are the uniaxial compressive strength of rock and rock mass, respectively.

In this study, the classification criteria [4] are further improved, aiming at accurately describing the severity of spalling, as shown in Table 2. The spalling severity is medium when the indicator σ_1/σ_c belongs to the interval [0.1, 0.2], and the risk levels are denoted as low or strong while the indicator is less than 0.1 or more than 0.2. The severity is medium D_i when it belongs to the interval [0.3, 0.5], while the interval [0, 0.3] and (0.5,∞) correspond to the risk levels low and strong,

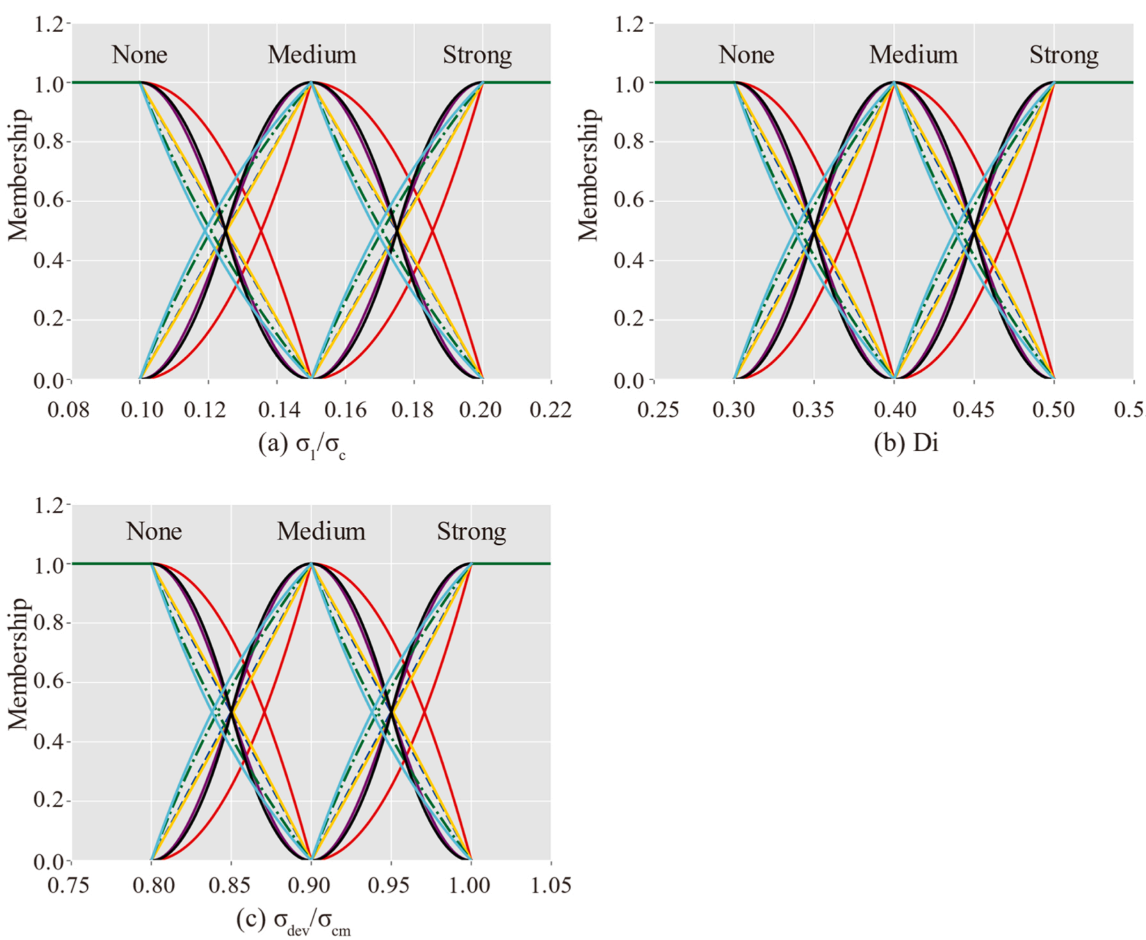

Fig. 3. Membership function of spalling.

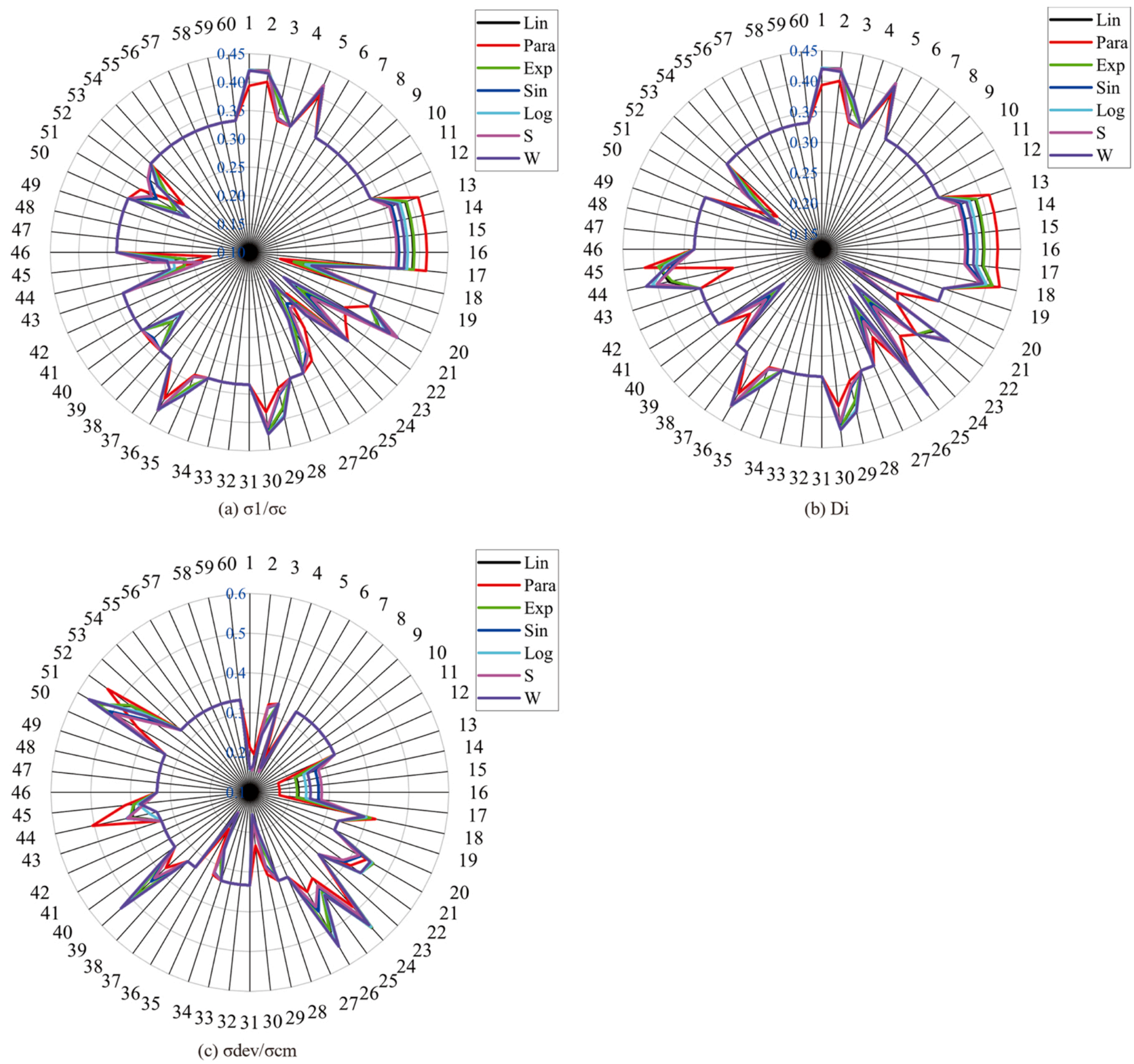

Fig. 4. The weights of indexes under different functions.

respectively. Similarly, for the indicator σ_{dev}/σ_{cm}, interval [0.8, 1] corresponds to the risk level medium, while the spalling grades low and strong connecting to the interval [0, 0.8] and (1, ∞), respectively.

3.2. Determination of membership integrating multiple functions

All the initial datasets and their classification criteria are obtained in Table 1 and Table 2. The core steps of the data-driven model constructed in this study include the construction of multiple membership functions, and the former tries to quantify the complex non-linear relationship of the index with different engineering backgrounds. At the same time, the latter focuses on highlighting the importance of the individual factor. There, seven membership functions are integrated into the unascertained model, including linear, parabolic, exponential, sine, logarithmic, S and Weibull functions, to measure the risk level of spalling. In Fig. 3, the shape of various functions is displayed vividly in the corresponding range.

$$\mu_i^{lin} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0.55 & 0.45 & 0 \end{bmatrix} \mu_i^{para} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0.798 & 0.202 & 0 \end{bmatrix} \mu_i^{\exp} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0.562 & 0.438 & 0 \end{bmatrix} \tag{20}$$

$$\mu_i^{\sin} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0.578 & 0.422 & 0 \end{bmatrix} \mu_i^{\log} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0.464 & 0.536 & 0 \end{bmatrix} \mu_i^{s} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0.595 & 0.405 & 0 \end{bmatrix} \tag{21}$$

$$\mu_i^{weibull} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0.427 & 0.573 & 0 \end{bmatrix} \tag{22}$$

3.3. Calculation of weight and composite measurement

To mine as much valuable information from the available dataset as possible and to guarantee the objectivity of index weight determination, the entropy-weight method describes the importance of individual indexes quantitatively. The calculation of index weight is based on the multiple index measurement matrix; by comparison, this method can directly highlight the spatial difference of samples compared to those calculated through the initial dataset. The weights of indexes are visualized in Fig. 4, e.g., the weights of sample 1 are {0.421, 0.422, 0.157}. The index weights of the individual samples are different, which is more feasible for on-site evaluation considering the uncertainty and

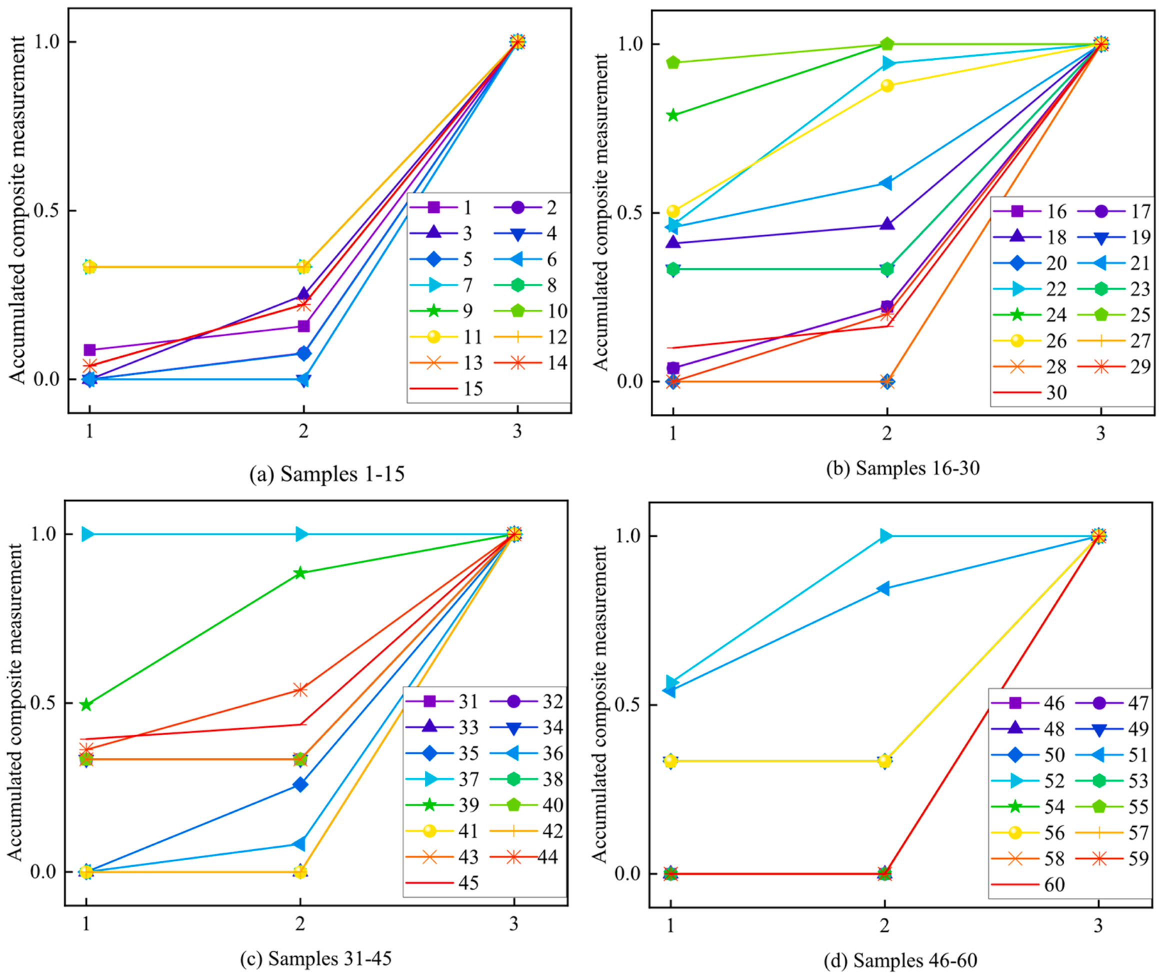

Fig. 5. Measurement vectors of samples.

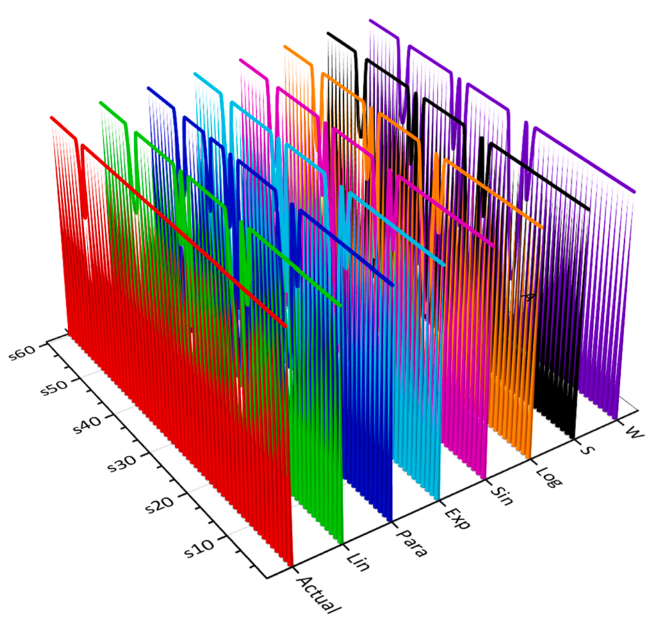

Fig. 6. Risk level of spalling calculated by seven functions.

complexity of the environment. For example, for samples 24, 26, 39, 44, 51 and 52, the role of σ_{dev}/σ_{cm} is more important than the other two indexes, while the three indexes are equally important for the remaining samples. Measurement vectors of samples are obtained according to Eq. (12), as shown in Fig. 5. The horizontal axe represents the risk levels (G1, G2 and G3) of spalling evaluation, respectively, while the vertical axe represents the accumulated measurement vectors.

3.4. *Spalling grade and scores*

Measurement vectors can investigate the membership of samples belonging to each grade. Then, to further explore the risk level of samples, a reliable identification principle R_η is used to classify the objectives based on measurement vectors ($R_\eta = 0.7$). Compared to the maximum measurement principle, the mentioned criterion can evaluate the risk level using the valuable information mined from the available dataset, although its selection process is somewhat subjective. Take sample 1 as an example, $0.086 + 0.071 + 0.843 = 1 > 0.7$, the risk level of sample 1 should be attributed to grade 3, i.e., strong. Similarly, the risk levels of the remaining samples are calculated according to Eq. (13). In Fig. 6, the X, Y and Z axes are the samples, functions, and classification results. The results of a large proportion of samples are consistent with the actual situation; simultaneously, the size of the dataset does not limit the performance of the established model. Using an unascertained measurement model is valid for rock mass spalling evaluation, which can offer a valuable basis for underground design and rock burst control.

A reliable identification principle is available for identifying the grade of spalling samples; however, it cannot highlight the difference between samples belonging to the same risk grade from the perspective of cost control. A sample score is used in this study for underground rock

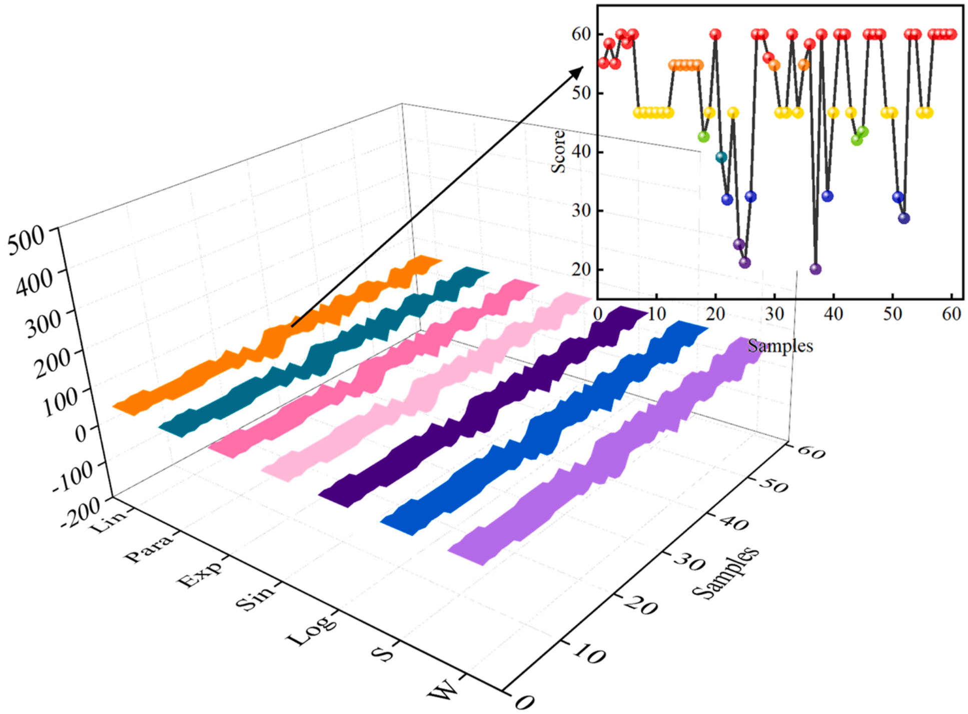

Fig. 7. Sample scores calculated by seven membership functions.

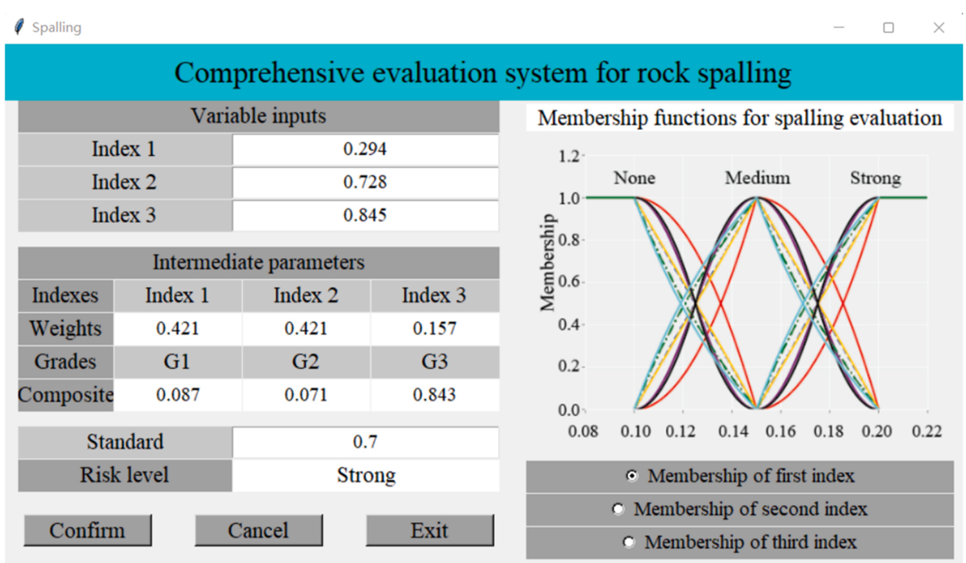

Fig. 8. Spalling evaluation GUI based on the unascertained measurement model.

mass spalling evaluation to analyze the risk and make cost-efficient plans. The risk level, i.e., “None”, “Weak”, and “Strong”, are assigned with a value, and then the sample score can be obtained according to Eq. (14). In this study, the larger the value, the higher the spalling risk. The quantitative score of samples calculated by different membership functions is visualized in Fig. 7. Each colour represents the risk score of samples calculated by a single function; meanwhile, the calculation of the linear function is displayed in an enlarged way. It can be seen that the difference in sample scores among various functions is small. The results are more reliable than those calculated by a single function.

In Fig. 7, the values of a considerably large proportion of samples more than 50, connecting to the high-risk level of spalling. Simultaneously, the risk control measures can focus on these high-risk level samples and areas, which is cost-efficient for underground hazard control and prevention.

This study uses the unascertained measurement model, integrating

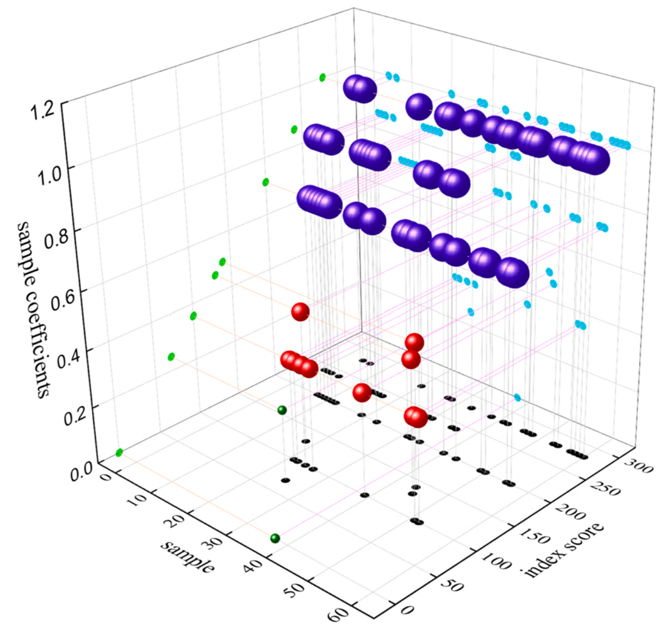

Fig. 9. Spalling degree obtained through empirical rating method.

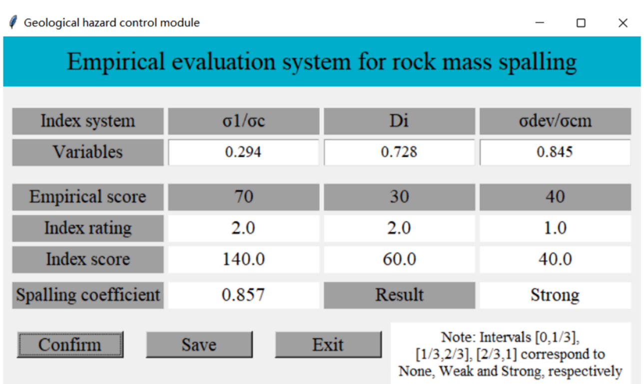

Fig. 10. Empirical method GUI for spalling evaluation.

membership functions and objective weights, to evaluate the rock mass spalling in underground infrastructure. The unascertained measurement model calculates the measurement vectors based on a couple of membership functions and investigates the index importance through entropy theory. These procedures are generally difficult for researchers from another area to understand. Meanwhile, it will take a lot of time and energy to evaluate rock spalling while datasets are large. Thus, a highly efficient and reliable GUI is developed to meet the requirements, as shown in Fig. 8.

The GUI is divided into four parts: variable inputs, intermediate parameters, evaluation results, and membership function visualization. Comprehensive evaluation regarding rock mass spalling in underground excavation zones refers to initial data collection and classification criteria; the former is determined in the parameters input module. At the same time, the latter has been fixed in the proposed model. In Fig. 8, module 1 is responsible for the parameter inputs corresponding to the evaluation indexes σ_1/σ_c, D_i and $\sigma_\mathrm{dev}/\sigma_{cm}$. For module 2, index weights and measurement vectors are highlighted, referring to Eqs. (11) and (12). Module 3 includes the reliable identification principle and classification results, and the former needs to be determined by the designer. Module 4 refers to the visualization of indexes for spalling evaluation. Operators easily obtain the membership degree of individual index by choosing the button corresponding to the index.

3.5. *Empirical rating methods for spalling evaluation*

The above section uses a data-driven model to evaluate spalling hazards in rock mass by objectively integrating multiple membership functions. Additionally, researchers' rich experience is vital for hazard evaluation at deeper ground, especially in situations where available information is not enough in the review. There, an empirical rating method is developed to evaluate the spalling degree, and its principle is clarified clearly in section 2.6. Take sample 1 as an example. The initial value of σ_1/σ_c, D_i and $\sigma_\mathrm{dev}/\sigma_{cm}$ are 0.294, 0.728 and 0.845, respectively. Then, the ratings of the three indexes above are denoted as 2, 2 and 1, respectively, referring to Table 2. In this study, empirical scores, i.e., the fixed initial empirical value of individual index given by the designer, are specified as {70, 30, 40}. Subsequently, the index and sample scores are obtained using Eqs. (15) and (16), through which the spalling risk coefficient can be calculated. There, the coefficient of sample 1 is 0.857, referring to Eq. (17), i.e., the spalling degree is considered “Strong”. Similarly, the calculation of the remaining samples is listed in Fig. 9. The X, Y, and Z axes are the samples, index score, and sample coefficients. Meanwhile, the sample size and colour correspond to the samples' evaluation results. It can be seen that a large proportion of samples are evaluated as “Strong”, consistent with the actual situation perfectly.

An empirical evaluation GUI has been developed to remove the complex calculation of the proposed empirical method and make it widely accepted by more practitioners, as we did in the data-driven evaluation module. In Fig. 10, the spalling coefficient and evaluation results are obtained directly after inputting the parameters of three evaluation indexes while the size of the dataset is large.

4. Discussion

Much research has been conducted on rock bursts from the perspective of physical models, mechanism analysis, computer science, etc. By comparison, little importance has been attached to such a geological hazard as spalling. The geological environment is considerably complex considering various operational and environmental factors, preventing us from comprehensively investigating/collecting the parameters regarding rock spalling. Thus, it is clear that unascertained measurement theory should prioritise spalling evaluation with its excellent performance on uncertainty information treatment. In the previous research, researchers tried to improve the model performance based on combination weight by integrating their advantages. This method can be considered one of the most efficient methods considering the role of the researcher's evaluation experience; however, it suffers from complex calculation and subjectivity while selecting the weight methods. Thus, the necessity and priority of this study can be summarized as follows:

1) Six non-linear functions are introduced to optimize the proposed model framework. Compared to the previous research using four membership functions, multiple functions can improve the ability to treat unascertained information.

2) A knowledge-based empirical method, i.e., spalling rating method, is proposed to analyze the risk of rock spalling quantitatively. In previous comprehensive evaluation studies, weight distribution for individual indexes was a common exploration to integrate practitioners' engineering knowledge and make up for conventional data-driven models' shortcomings. There, the principle of the empirical model is easy to understand and can provide valuable information for underground design. More importantly, the classification performance of the proposed empirical is as good as the data-driven model, which can further prove the efficiency of the initial index score we set.
3) Seven membership functions are utilized simultaneously to measure the unascertained information and to mutually verify the classification performance of individual functions, effectively removing the negative impact of random details on classification results caused by single functions. Additionally, data-driven and knowledge-based empirical models can extract valuable information and integrate researchers' rich experience into engineering valuation. In this case, the evaluation results are more reliable and feasible than those obtained by a single objective/subjective model, which can balance the calculation objectivity and the reliability of results. Finally, the GUIs are developed for the mentioned models, making the calculation more time-efficient.

5. Conclusion

An improved unascertained measurement model integrating seven membership functions is used to evaluate the risk of rock spalling in underground infrastructure. Simultaneously, a knowledge-based empirical method for spalling evaluation is proposed in this study. A total of 60 data groups is used to validate the performance of constructed subjective and objective models. The main conclusions are highlighted as follows:

1) Seven membership functions are integrated into the unascertained measurement model, aiming to improve the proposed data-driven model's application scenarios. The entropy-weight method makes weight calculation more objective, while the reliable identification principle can largely prevent the loss of valuable decision-making information. The classification accuracy of the proposed model is 88 %, which can be accepted for geological hazard prevention considering the uncertainty and complexity of the geo-environment.
2) An empirical method, i.e., rock spalling coefficients, is proposed to evaluate the spalling based on the collected dataset referring to the fixed index score. The classification accuracy is 82 %, while the initial index score is 70, 30 and 40, respectively.
3) Despite their excellent performance, the proposed models still cannot meet the on-site safety requirements, but they can be considered a valid exploitation for hazard assessment. Future research should investigate more highly efficient combination models to treat various uncertainty information at once.

CRediT authorship contribution statement

Chen Chao: Writing – original draft, Visualization, Validation, Methodology, Formal analysis, Data curation. **Yao Shibing:** Writing – review & editing, Formal analysis. **Zhou Jian:** Writing – review & editing, Validation, Supervision, Methodology, Funding acquisition, Conceptualization.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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References

[1] J. Zhou, C. Chen, K. Du, D. Jahed Armaghani, C. Li, A new hybrid model of information entropy and unascertained measurement with different membership functions for evaluating destressability in burst-prone underground mines, Eng. Comput. 38 (2022) 381–399.
[2] C.D. Martin, P.K. Kaiser, D.R. McCreath, Hoek-Brown parameters for predicting the depth of brittle failure around tunnels, Can. Geotech. J. 36 (1999) 136–151.
[3] J. Zhou, X. Li, H.S. Mitri, Evaluation method of rockburst: state-of-the-art literature review, Tunn. Undergr. Space Technol. 81 (2018) 632–659.
[4] K.H. Lee, J.H. Bang, I.M. Lee, Y.-J. Shin, Use of fuzzy probability theory to assess spalling occurrence in underground openings, Int. J. Rock. Mech. Min. Sci. 64 (2013) 60–67.
[5] C. Wang, D.D. Tannant, Rock fracture around a highly stressed tunnel and the impact of a thin tunnel liner for ground control, Int. J. Rock. Mech. Min. Sci. 41 (2004) 676–683.
[6] J. Zhou, Y. Zhang, C. Li, H. He, X. Li, Rockburst prediction and prevention in underground space excavation, Undergr. Space 14 (2024) 70–98.
[7] W.C. Zhu, Z.H. Li, L. Zhu, C.A. Tang, Numerical simulation on rockburst of underground opening triggered by dynamic disturbance, Tunn. Undergr. Space Technol. 25 (2010) 587–599.
[8] J. Zhou, X. Li, X. Shi, Long-term prediction model of rockburst in underground openings using heuristic algorithms and support vector machines, Saf. Sci. 50 (2012) 629–644.
[9] J. Zhou, X. Li, H.S. Mitri, Classification of rockburst in underground projects: comparison of ten supervised learning methods, J. Comput. Civ. Eng. 30 (2016) 04016003.
[10] J. Zhou, X. Li, H.S. Mitri, Comparative performance of six supervised learning methods for the development of models of hard rock pillar stability prediction, Nat. Hazards 79 (2015) 291–316.
[11] J. Zhou, C. Chen, C. Wei, K. Du, An improved connection cloud model of an updated database: a multicriteria uncertainty model for coal burst liability evaluation, Nat. Resour. Res. 31 (2022) 1687–1704.
[12] L. Xu, F. Gong, Z. Liu, Experiments on rockburst proneness of pre-heated granite at different temperatures: insights from energy storage, dissipation and surplus, J. Rock. Mech. Geotech. Eng. 14 (2022) 1343–1355.
[13] F. Gong, Z. He, Q. Jiang, Internal mechanism of reducing rockburst proneness of rock under high stress by real-time drilling pressure relief, Rock. Mech. Rock. Eng. 55 (2022) 5063–5081.
[14] X. Li, J. Zhou, S. Wang, B. Liu, Review and practice of deep mining for solid mineral resources, Zhongguo Youse Jinshu Xuebao/Chin. J. Nonferrous Met. 27 (2017) 1236–1262.
[15] H. Farhadian, A new empirical chart for rockburst analysis in tunnelling: Tunnel rockburst classification (TRC), Int. J. Min. Sci. Technol. 31 (2021) 603–610.
[16] K.K. Panthi, Review on the prevailing methods for the prediction of potential rock burst / rock spalling in tunnels. Norsk Forening for Fjellsprengningsteknikk, Norsk Bergmekanikkgruppe og Norsk Geoteknisk Forening Oslo, 2017.
[17] J.P. Harrison, J.A. Hudson, Incorporating parameter variability in rock mechanics analyses: fuzzy mathematics applied to underground rock spalling, Rock. Mech. Rock. Eng. 43 (2010) 219–224.
[18] C. Chen, J. Zhou, T. Zhou, W. Yong, Evaluation of vertical shaft stability in underground mines: comparison of three weight methods with uncertainty theory, Nat. Hazards 109 (2021) 1457–1479.
[19] J. Zhou, C. Chen, M. Wang, M. Khandelwal, Proposing a novel comprehensive evaluation model for the coal burst liability in underground coal mines considering uncertainty factors, Int. J. Min. Sci. Technol. 31 (2021) 799–812.
[20] J. Zhou, C. Chen, D.J. Armaghani, S. Ma, Developing a hybrid model of information entropy and unascertained measurement theory for evaluation of the excavatability in rock mass, Eng. Comput. 38 (2022) 247–270.
[21] Y. Shi, X. Shi, W. Ding, Comprehensive safety evaluation of huangjindong tailing pond based on improved entropy weight method-unascertained measure model, Gold. Sci. Technol. 29 (2021) 155–163.
[22] J. Zhou, C. Chen, M. Khandelwal, M. Tao, C. Li, Novel approach to evaluate rock mass fragmentation in block caving using unascertained measurement model and information entropy with flexible credible identification criterion, Eng. Comput. 38 (2022) 3789–3809.
[23] J. Zhou, X. Li, Integrating unascertained measurement and information entropy theory to assess blastability of rock mass, J. Cent. South Univ. 19 (2012) 1953–1960.
[24] X. Ding, X. Tian, J. Wang, A comprehensive risk assessment method for hot work in underground mines based on G1-EWM and unascertained measure theory, Sci. Rep. 14 (2024) 6063.
[25] B.D. Hall, Object-oriented software for evaluating measurement uncertainty, Meas. Sci. Technol. 24 (2013) 055004.
[26] Morawski, Application-oriented meta-model of measurement uncertainty, Measurement 225 (2024) 114044.
[27] Q. Cheng, Attribute sets and attribute synthetic assessment system, Syst. Eng. - Theory Pract. 17 (1997) 2–9.

[28] Q. Cheng, Attribute recognition theoretical model with application, Acta Sci. Nat. Univ. Pekin. 33 (1997) 13–19.

[29] N. Li, X. Feng, R. Jimenez, Predicting rock burst hazard with incomplete data using Bayesian networks, Tunn. Undergr. Space Technol. 61 (2017) 61–70.

[30] C.D. Martin, P.K. Kaiser, D.R. McCreath, Hoek-Brown parameters for predicting the depth of brittle failure around tunnels, Can. Geotech. J. 36 (1999) 136–151.

[31] L. Weng, L. Huang, A. Taheri, X. Li, Rockburst characteristics and numerical simulation based on a strain energy density index: a case study of a roadway in Linglong gold mine, China, Tunn. Undergr. Space Technol. 69 (2017) 223–232.

[32] J. Zhou, Strain burst prediction and spalling depth estimation using supervised learning methods, Central south university, 2015.

[33] M. Cai, P.K. Kaiser, Y. Tasaka, T. Maejima, H. Morioka, M. Minami, Generalized crack initiation and crack damage stress thresholds of brittle rock masses near underground excavations, Int. J. Rock. Mech. Min. Sci. 41 (2004) 833–847.

Jian Zhou obtained his BSc (2008) and PhD (2015) from Central South University (CSU), China, and served as a visiting scholar at the Mine Design Laboratory at McGill University, Canada, from 2013 to 2014. Currently, he is a professor in the School of Resources and Safety Engineering at CSU. He is recognized as a National High-Level Young Talent and Distinguished Young Scholar by the Hunan Province. His current research interests include the prediction and mitigation of geological and geotechnical hazards, with a focus on applying predictive models to rock mechanics and mining engineering. Dr. Zhou is a Highly Cited Researcher in the Cross-Field category (Clarivate), a Highly Cited Chinese Researcher in the field of mining engineering (Elsevier), and ranked among the top 2 % of scientists globally (Career-long and single-year impact) by Stanford University. He has published over 100 papers in international journals on mining and geotechnical issues, with citations totaling 14,000 and an H-index of 67. Additionally, he has received China's 100 Most Influential International Academic Papers Award, as well as Best Paper Awards from the Journal of Rock Mechanics and Geotechnical Engineering and the Journal of Central South University.